CALCULATION OF PARAMETERS OF INTERNAL HEAT AND MASS TRANSFER DURING DRYING OF FIBER MATERIAL

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Using arguments and characteristic functions of the thermodynamics of irreversible processes, calculations are made of parameters of internal heat and mass transfer during drying of fiber material in melted paraffin.

During drying of fiber material in melted paraffin a considerable excess pressure P is generated, and the internal transfer of heat and moisture is described by the system of equations [1]

$$\frac{\partial T}{\partial \tau} = \left(a + \frac{\varepsilon r}{c} a_m\right) \nabla^2 T + \frac{\varepsilon r}{c} a_m \nabla^2 U + \varepsilon r \frac{a_m}{c} \delta_p \nabla^2 P,$$

$$\frac{\partial U}{\partial \tau} = a_m \nabla^2 U + a_m \delta \nabla^2 T + \frac{k_f}{c_f P} \nabla^2 P,$$

$$\frac{\partial P}{\partial \tau} = -\frac{\varepsilon a_m}{c_f} \nabla^2 U - \frac{\varepsilon a_m \delta}{c_f} \nabla^2 T + \left(a_f - \frac{\varepsilon a_m}{c_f} \delta_p\right) \nabla^2 P. \qquad (1)'$$

This description of the process requires calculation of nine parameters. However, the vapor density per unit volume

$$\varepsilon U \gamma_0 = \rho_v \qquad (2)$$

is related to the pressure and temperature of the vapor by the equation of state, from which the pressure may always be found as a function of humidity and temperature:

$$P = f(U, t). \qquad (3)$$

Substituting this expression into (1), we may obtain a system of transfer equations, which in the linear approximation has the form

$$\frac{\partial T}{\partial \tau} = a \nabla^2 T + \frac{\varepsilon r}{c} \frac{\partial U}{\partial \tau},$$

$$\frac{\partial U}{\partial \tau} = a_m \nabla^2 U + a_m \delta \nabla^2 T, \qquad (4)$$

but here the transfer coefficients are equivalent; they take the molar mass transfer into account.

To determine these equivalent values of the transfer parameters, use is made of the method of characteristic functions of the thermodynamics of irreversible processes [2]. In this method of calculation, we must first find the experimental functions

$$Z_a = \frac{\partial T(N_0, H)}{\partial H} \Big/ U_v'(H) \qquad (5)$$

and

$$\xi_a = [T(N_1, H) - T(N_0, H)]/U_v'(H), \qquad (6)$$

which allow us to find the thermal diffusivity of the material

$$a = \frac{N_1^2 - N_0^2}{2k} \frac{R^2}{\tau_0} \left[\frac{\partial Z_a}{\partial \xi_a}\right]_{N_1, N_0} \qquad (7)$$

and the parametric group

$$\varepsilon \mathrm{Ko} = Z_a - \xi_a \left[\frac{\partial Z_a}{\partial \xi_a}\right]_{N_1, N_0}. \qquad (8)$$

The dimensionless modifications of the test values of integral humidity U_V, local temperature T, and temperature difference

$$\Delta = T(N_1, H) - T(N_0, H) \qquad (9)$$

are given in Table 1. Also given is the dimensionless drying rate $U_V'(H)$ and the dimensionless heating rate $T'(N_0, H)$ as a function of dimensionless time

$$H = \tau/100. \qquad (10)$$

The humidity is normalized to 200%:

$$U_v(H) = U_v(\tau)/200. \qquad (11)$$

The dimensionless temperature is

$$T = [t(N, \tau) - 18]/100. \qquad (12)$$

The groups εKo and Pn are normalized to the same scale values of humidity U_0 = 200% and t_0 = 100° C.

The coordinate N in (7) is taken as the ratio of the distance from the axis of the fibrous tube to a fixed internal radius R = 17 mm. The coordinates of points at which thermocouples were embedded, $N_1 = 1.221$ and $N_0 = 1.145$, differed so little that the temperature and humidity fields between these points may be considered to be planes and k may be taken equal to 1.

Table 2 shows the functions Z_a and ξ_a as a function of dimensionless time τ. They are shown for the period of increasing drying rate H ∈ [0–0.12], and for two sections of the period of falling drying rate, H ∈ [0.40–0.68] and H ∈ [0.78–1.10].

During time H ∈ [0.12–0.18] the heating rate $T'(N_0, H)$, the temperature drop Δ, and the drying rate U_V' differ very little from their measured mean values. The derivatives of these quantities are comparable with their errors [3], and therefore under the existing experimental technique, the parameters of internal transfer cannot be calculated. Moreover, and

Table 1

Experimental Values and Their Derivatives

H	$U_v(H)$	$T(N_0, H)$	Δ	$U'_v(H)$	$T'(N_0, H)$
0.00	1.035	0.000	0.000	−0.373	28.400
0.02	1.025	0.490	0.170	−1.938	18.650
0.04	0.953	0.730	0.080	−3.395	9.710
0.06	0.864	0.800	0.030	−4.250	3.730
0.08	0.776	0.816	0.014	−4.410	1.040
0.10	0.688	0.820	0.010	−4.280	0.501
0.12	0.600	0.822	0.008	−3.860	0.200
0.14	0.524	0.822	0.009	−3.170	0.100
0.16	0.472	0.822	0.009	−2.820	0.000
0.18	0.430	0.822	0.010	−2.115	0.000
0.20	0.393	0.822	0.012	−1.745	0.000
0.22	0.363	0.823	0.013	−1.482	0.040
0.24	0.334	0.824	0.015	−1.265	0.040
0.26	0.313	0.824	0.017	−1.080	0.040
0.28	0.293	0.824	0.018	−0.930	0.040
0.30	0.277	0.825	0.019	−0.811	0.040
0.32	0.262	0.826	0.021	−0.725	0.040
0.34	0.249	0.827	0.023	−0.656	0.040
0.36	0.236	0.828	0.024	−0.598	0.040
0.38	0.225	0.829	0.026	−0.545	0.040
0.40	0.214	0.830	0.027	−0.510	0.120
0.42	0.205	0.832	0.024	−0.426	0.220
0.44	0.197	0.838	0.022	−0.397	0.295
0.46	0.189	0.845	0.022	−0.371	0.320
0.48	0.183	0.852	0.022	−0.358	0.370
0.50	0.175	0.859	0.020	−0.313	0.380
0.52	0.170	0.868	0.019	−0.280	0.390
0.54	0.164	0.875	0.017	−0.265	0.385
0.56	0.159	0.883	0.017	−0.240	0.365
0.58	0.154	0.890	0.016	−0.230	0.336
0.60	0.150	0.896	0.015	−0.220	0.370
0.62	0.147	0.902	0.015	−0.210	0.390
0.64	0.141	0.914	0.015	−0.201	0.350
0.66	0.138	0.920	0.014	−0.194	0.370
0.68	0.133	0.922	0.013	−0.181	0.100
0.70	0.130	0.922	0.013	−0.170	0.057
0.72	0.127	0.922	0.013	−0.150	0.000
0.74	0.124	0.922	0.013	−0.150	0.000
0.76	0.121	0.922	0.016	−0.143	0.000
0.78	0.118	0.922	0.016	−0.136	0.050
0.80	0.115	0.924	0.016	−0.127	0.145
0.82	0.113	0.928	0.016	−0.122	0.200
0.84	0.111	0.932	0.016	−0.116	0.230
0.86	0.109	0.938	0.015	−0.106	0.250
0.88	0.105	0.942	0.014	−0.100	0.250
0.90	0.104	0.948	0.014	−0.0982	0.250
0.92	0.103	0.952	0.014	−0.0925	0.250
0.94	0.101	0.958	0.014	−0.0877	0.230
0.96	0.099	0.964	0.014	−0.0845	0.300
0.98	0.097	0.970	0.014	−0.0834	0.310
1.00	0.096	0.976	0.012	−0.0773	0.315
1.02	0.094	0.983	0.012	−0.0725	0.305
1.04	0.093	0.989	0.012	−0.0675	0.285
1.06	0.091	0.994	0.012	−0.0652	0.215
1.08	0.090	0.997	0.010	−0.0625	0.290
1.10	0.089	1.000	0.010	−0.0625	0.190

Table 2

Arguments of the Characteristic Functions and Parameters of Internal Transfer εKo and λ as a Function of Temperature and Humidity

Time τ,min	T, °K	U, kg wet/ /kg dry	Z_a	ξ_a	Numerical expressions for the quantities	ε Ko	λ, W/m · · degree
2	348.7	2.05	—9.62	—0.0877	$\partial Z_a/\partial \tau = 31.5$ $\partial \xi_a/\partial \tau = 0.246$ $\partial Z_a/\partial \xi_a = 127.0$ $a = 20 \cdot 10^{-4}$ m²/hr	1.38	6.891
4	368.2	1.86	—2.78	—0.0236		0.22	6.469
6	372.7	1.73	—0.88	—0.0070		0.01	5.057
8	373.5	1.55	—0.24	—0.0036		0.22	5.444
10	373.7	1.38	—0.12	—0.0024		0.19	4,932
12	373.7	1.20	—0.052	—0.0016		0.15	4.419
40	375.6	0.43	—0.235	—0.0530	$\frac{\partial Z_a}{\partial \tau} = -6.194$ $\frac{\partial \xi_a}{\partial \tau} = -0.0736$ $\partial Z_a/\partial \xi_a = 84.0$ $a = 13.1 \cdot 10^{-4}$ m²/hr	4.22	1.426
42	375.6	0.41	—0.516	—0.0563		4.21	1.390
44	376.1	0.39	—0.743	—0.0555		3.97	1.331
46	376.8	0.37	—0.863	—0.0595		4.14	1.305
48	377.5	0.36	—1.034	—0.0615		4.13	1.276
50	378.1	0.35	—1.214	—0.0640		4.16	1.257
52	379.0	0.34	—1.393	—0.0670		4.24	1.217
54	380.1	0.33	—1.453	—0.0642		3.94	1.196
56	380.4	0.32	—1.521	—0.0710		4.44	1.181
58	381.0	0.31	—1.457	—0.0696		4.39	1.164
60	381.6	0.30	—1.682	—0.0682		4.05	1.148
62	382.2	0.29	—1.857	—0.0715		4.15	1.133
64	382.9	0.28	—1.750	—0.0745		4.51	1.125
66	383.9	0.27	—1.907	—0.0723		4.17	1.119
68	384.1	0.26	—0.552	—0.0720		5.50	1.112
78	384.1	0.236	—0.368	—0.118	$\partial Z_a/\partial \tau = -24$ $\partial \xi_a/\partial \tau = -0.329$ $\partial Z_a/\partial \xi_a = 73.2$ $a = 11.39 \cdot 10^{-4}$ m²/hr	8.31	0.921
80	384.4	0.230	—1.142	—0.126		8.08	0.911
82	384.8	0.226	—1.639	—0.131		7.95	0.902
84	385.2	0.221	—1.983	—0.138		8.12	0.896
86	385.8	0.218	—2.358	—0.141		7.96	0.890
88	386.1	0.210	—2.500	—0.140		7.75	0.881
90	386.7	0.208	—2.546	—0.143		7.92	0.875
92	387.1	0.205	—2.703	—0.151		8.25	0.869
94	387.7	0.201	—2.623	—0.160		9.01	0.863
96	388.3	0.198	—3.550	—0.166		8.60	0.857
98	388.9	0.194	—3.717	—0.174		9.02	0.851
100	389.4	0.191	—4.075	—0.155		7.27	0.846
102	390.1	0.187	—4.207	—0.166		7.94	0.840
104	390.7	0.185	—4.222	—0.178		8.81	0.836
106	391.2	0.182	—3.298	—0.184		10.17	0.832
108	391.4	0.180	—4.640	—0.176		8.24	0.828
110	391.7	0.178	—3.040	—0.160		8.67	0.824

Table 3

Values of the Argument Z_p and the Parameters a_m, Lu, Pn, δ

Time τ, min	Z_p	Numerical expressions for the quantities	Lu	$a_m \cdot 10^4$ m²/hr
78	7.85	$\frac{\partial Z_p}{\partial H} = 18.90$ Pn = 0.785 $\delta = 1.5 \frac{1}{\text{degree}}$	0.935	10.650
80	8.95		0.572	6.515
82	9.10		0.642	7.312
84	9.40		0.686	7.814
86	10.20		0.477	5.433
88	10.40		0,427	4.863
90	10.43		0.451	5.137
92	10.95		0.440	5.012
94	11.28		0.480	5.467
96	11.53		0.505	5.752
98	11.48		0.675	7.688
100	12.18		0.305	3.474
102	12.80		0.308	3.508
104	13.43		0.321	3.656
106	13.80		0.319	3.633
108	14.20		0.256	2.916
110	14.05		0.128	1.458

this is particularly important, during time $H \in [0.18-0.10]$, the phase transition surface moves from the point $N_1 = 1.221$ to the point $N_0 = 1.145$. For such time intervals Luikov's systems of equations [1], and as a result the expansions in terms of the experimental functions of Temkin [3], do not occur, since the points N_1 and N_0 are located in zones where there are different forms of bonds between moisture and material. For this reason the transfer parameters could not be calculated over the major part of the interval $H \in$ $\in [0.12-0.40]$.

Transition of moisture from the capillary-locked to the capillary-free state between points N_1 and N_0 takes place during time $H \in [0.68-0.78]$. This means physically that there is a complete cessation of migration of capillary-bound moisture in the condensed phase. The transition of moisture from the capillary-locked to the capillary-free state results in its movement only in the form of vapor. This corresponds to a sharp increase in the group εKo, since $\varepsilon = 1$, and a decrease in thermal conductivity (Table 2). The comparatively large value of εKo at the beginning of the process is due to the energy expended in forming a capillary surface of evaporation, i.e., working against the forces of surface tension. The very small value of εKo near $H = 0.04$ may be explained by the fact that in this time the moisture moves only in the condensed phase, and the group $\varepsilon \cong 0$.

To calculate the inertia Lu and thermal diffusion Pn numbers we must first find the quantity

$$Z_p = -\frac{2k(k+2)a\tau_0}{[(k+2)-k]R^2}\,\frac{U_v(H)}{U_v'(H)}, \tag{13}$$

which is given in Table 3. In accordance with [2] the inertia number is found from the formula

$$\mathrm{Lu}^{-1} = Z_p - \xi_a\left[\frac{\partial Z_p}{\partial \xi_a}\right]_{(N_0,\,N_1)}, \tag{14}$$

and the thermal diffusion number from the formula

$$\mathrm{Lu}^{-1} + \varepsilon\,\mathrm{KoPn} = Z_p - Z_a\left[\frac{\partial Z_p}{\partial Z_a}\right]_{N_0,\,N_1}. \tag{15}$$

Their values are shown in Table 3.

From a comparison of the computed equivalent values of the coefficients of internal moisture transfer with the same coefficients found for purely molecular transfer [4], it may be seen that the new transfer coefficients (equivalent) are several orders larger. This indicates that the main motive force of heat and mass transfer in high-intensity drying is the excess vapor pressure gradient.

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